Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Graph Theory

Subject Code: 5SC04GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 18/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) Define: Incidence matrix **(02)**

b) Draw a simple graph with 6 vertices and 12 edges. **(02)**

c) Define: Arborescence **(02)**

d) Find the in-degree and out-degree of complete symmetric digraph with 4 vertices. **(01)**

Q-2 Attempt all questions **(14)**

a) State and prove necessary and sufficient condition for the graph is disconnected. **(07)**

b) Define the following: **(05)**

1) Bipartite graph 2) Eccentricity 3) Cut-set 4) Circuit matrix

c) Prove that number of edges in complete graph is $\frac{n(n-1)}{2}$. **(02)**

OR

Q-2 Attempt all questions **(14)**

a) Let G be a tree with n vertices then prove that G has $n-1$ edges. **(07)**

b) Answer the following questions from the following graph **(05)**

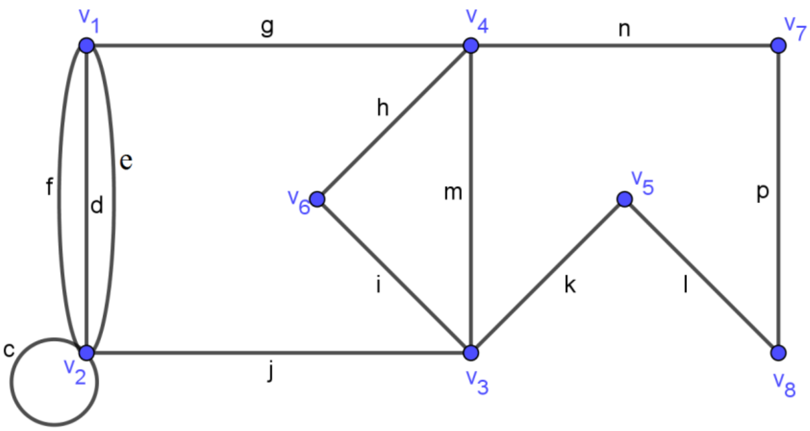


Figure – 1

i) Write one Spanning tree.

ii) Write one fundamental circuit w.r.t. i).

iii) Write adjacency matrix.

iv) Write one closed walk of length 13.

c) Verify first theorem of graph theory for figure-1. **(02)**

Q-3 Attempt all questions **(14)**

a) Draw a diagraph and construct longest circular sequence of 1's and 0's such that no subsequence of 4 bits appears more than once in the sequence. **(05)**

b) From the following adjacency matrix draw the diagraph G. Also find X^4 and hence find the directed edge sequence of length four from v_1 to v_3. **(05)**

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

c) Define the following with examples: **(04)**

1) Symmetric digraph 2) condensation

OR

Q-3 Attempt all questions **(14)**

a) State and prove Teleprinter's problem. **(07)**

b) If G be a digraph then prove that determinant of every square sub-matrix of $A(G)$ is 1,-1 or 0. **(03)**

c) Define the following with examples: **(04)**

1) Strong component 2) Euler digraph

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) Define: Saturated vertices **(02)**

b) Define: Symmetric difference of matching **(02)**

c) Find the minimum chromatic number of any graph which contains a triangle. **(01)**

d) Find chromatic number of G if $E \neq \phi$ in any graph G. **(01)**

e) Define: Domination number **(01)**

Q-5 Attempt all questions **(14)**

a) Let G be a simple graph with n vertices and $d(v) \geq \frac{n}{2}$, for $\forall v \in G$ then G is a Hamiltonian graph, where $n \geq 3$. **(07)**

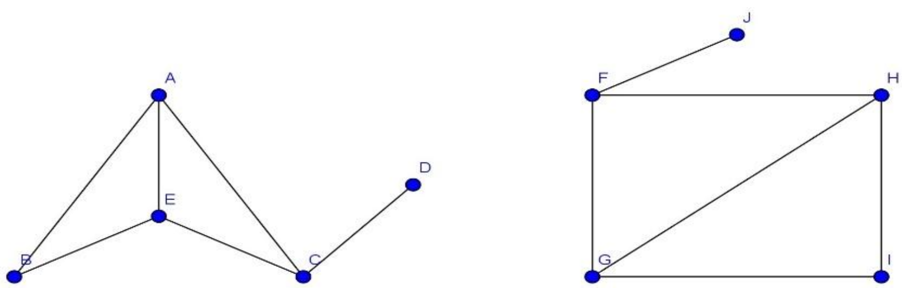
b) Define chromatic polynomial and also find it for C_4. **(07)**

OR

Q-5 Attempt all questions **(14)**

a) Prove that the vertices of every planner graph can be properly colored with 5 colors. **(09)**

b) Show that the following graphs are isomorphic. **(05)**



G_1 G_2

Figure – 3

Q-6 Attempt all questions **(14)**

a) State and prove Hall's theorem. **(10)**

b) Define the following with graph and also find the minimum size of vertex cover and edge cover of that graph. **(04)**

1) Vertex cover 2) Edge cover

OR

Q-6 Attempt all Questions **(14)**

a) State and prove Min-Max theorem. **(10)**

b) Answer the following questions from the following graph **(04)**

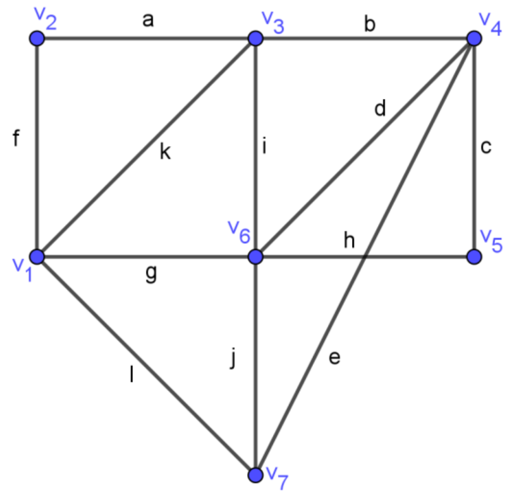


Figure – 4

i) Find a perfect matching and a maximum matching.

ii) Find one M-augmenting path and M-alternating path.